Regras de Derivação

① $f(x) = -\frac{1}{2}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 + \frac{1}{4}$

$f'(x) = -\frac{1}{\cancel{2}} \cdot 4 \cdot x^3 + \frac{2}{\cancel{3}} \cdot 3x^2 - \frac{1}{\cancel{2}} \cdot 2x + 0$

$f'(x) = -2x^3 + 2x^2 - x$

② $f(x) = x^2 + \sqrt{x}$

$f(x) = x^2 + x^{1/2}$

$f'(x) = 2x + \frac{1}{2}x^{\frac{1}{2}-1}$

$f'(x) = 2x + \frac{1}{2} \cdot x^{-1/2}$

$f'(x) = 2x + \frac{1}{2x^{1/2}} \Rightarrow 2x + \frac{1}{2\sqrt{x}}$

③ $f(x) = \overset{u}{x^3} \cdot \overset{v}{\cos x}$

$(u \cdot v)' = u' \cdot v + u \cdot v'$

$= (x^3)' \cdot (\cos x) + x^3 \cdot (\cos x)'$

$= 3x^2 \cdot \cos x + x^3 \cdot (-\operatorname{sen} x)$

$= 3x^2 \cdot \cos x - x^3 \cdot \operatorname{sen} x$

4) $f(x) = \underbrace{x^3}_{u} \cdot \underbrace{(2x^2 - 3x)}_{v}$

$(u \cdot v)' = u'v + u \cdot v'$

$= (x^3)' \cdot (2x^2 - 3x) + x^3 \cdot (2x^2 - 3x)'$

$= 3x^2 \cdot (2x^2 - 3x) + x^3 \cdot (4x - 3)$

$= 6x^4 - 9x^3 + 4x^4 - 3x^3$

$= 10x^4 - 12x^3$

5) $f(x) = \dfrac{2x+5}{4x} = \dfrac{u}{v}$

$\left(\dfrac{u}{v}\right)' = \dfrac{u'v - uv'}{v^2}$

$= \dfrac{(2x+5)' \cdot 4x - (2x+5) \cdot (4x)'}{(4x)^2}$

$= \dfrac{2 \cdot 4x - (2x+5) \cdot 4}{16x^2}$

$= \dfrac{\cancel{8x} - \cancel{8x} - 20}{16x^2}$

$= -\dfrac{20^{:4}}{16x^2{}_{:4}} = -\dfrac{5}{4x^2}$

⑥ $f(x) = \left(\frac{2}{5}\right)^x = a^u$

$(a^u)' = u' \cdot a^u \cdot (\ln a)$

$\left(\frac{2}{5}^x\right)' = x' \cdot \left(\frac{2}{5}\right)^x \cdot \ln\left(\frac{2}{5}\right)$

$= 1 \cdot \left(\frac{2}{5}\right)^x \cdot \ln\left(\frac{2}{5}\right) = \underline{\left(\frac{2}{5}\right)^x \cdot \ln\left(\frac{2}{5}\right)}$

⑦ $f(x) = 2^{3x-1} \Rightarrow a^u$

$(a^u)' = u' \cdot a^u \cdot (\ln a)$

$(2^{3x-1})' = (3x-1)' \cdot 2^{3x-1} \cdot \ln 2$

$= 3 \cdot 2^{3x-1} \cdot \ln 2$

$= \underline{2^{3x-1} \cdot 3 \ln 2}$

⑧ $f(x) = 3^x \Rightarrow a^u$

$(a^u)' = u' \cdot a^u \cdot \ln a$

$(3^x)' = x' \cdot 3^x \cdot \ln 3$

$= 1 \cdot 3^x \cdot \ln 3$

$= \underline{3^x \cdot \ln 3}$

(9) $f(x) = \text{sen}(x^2) = \text{sen } u$

$(\text{sen } u)' = u' . \cos u$

$= (x^2)' . \cos(x^2)$

$= \underline{2x . \cos(x^2)}$

(10) $f(x) = \cos\left(\frac{1}{x}\right) = \cos u$

$(\cos u)' = u' . (-\text{sen } u)$

$= \left(\frac{1}{x}\right)' . \left[-\text{sen}\left(\frac{1}{x}\right)\right]$

$= (1 . x^{-1}) . \left[-\text{sen}\left(\frac{1}{x}\right)\right]$

$= -1 . x^{-2} . \left[-\text{sen}\left(\frac{1}{x}\right)\right]$

$= -\frac{1}{x^2} . \left[-\text{sen}\left(\frac{1}{x}\right)\right]$

$= \underline{\underline{\frac{1}{x^2} . \text{sen}\left(\frac{1}{x}\right)}}$

(11) $f(x) = (\underbrace{x^2 + 5x + 2}_{u})^7 - u^7$

Regra da cadeia.

$\Rightarrow f'(u) \cdot u'(x)$

$= (u^7)' \cdot u'$

$= 7 \cdot u^6 \cdot u'$

$= 7 \cdot (x^2 + 5x + 2)^6 \cdot (x^2 + 5x + 2)'$

$= 7 \cdot (x^2 + 5x + 2)^6 \cdot (2x + 5)$

(12) $f(x) = \left(\dfrac{3x + 2}{2x + 1}\right)^5 \Rightarrow u^5$

$= f'(u) \cdot u'(x)$

$= (u^5)' \cdot u'(x)$

$= 5 \cdot u^4 \cdot u'(x)$

$= 5 \cdot \left(\dfrac{3x + 2}{2x + 1}\right)^4 \cdot \left[\dfrac{(3x + 2)' \cdot (2x + 1) - (3x + 2) \cdot (2x + 1)'}{(2x + 1)^2}\right]$

$= 5 \cdot \left(\dfrac{3x + 2}{2x + 1}\right)^4 \cdot \left[\dfrac{3 \cdot (2x + 1) - (3x + 2) \cdot 2}{(2x + 1)^2}\right]$

$= 5 \cdot \left(\dfrac{3x + 2}{2x + 1}\right)^4 \cdot \left[\dfrac{\cancel{6x} + 3 - \cancel{6x} - 4}{(2x + 1)^2}\right]$

$= 5 \left(\dfrac{3x + 2}{2x + 1}\right)^4 \cdot \left(\dfrac{-1}{(2x + 1)^2}\right)$

(13) $f(x) = \frac{1}{3}(\underbrace{2x^5 + 6x^{-3}}_{u})^5 = \frac{1}{3}u^5$

$= f'(u) \cdot u'(x)$

$= \frac{1}{3}(u^5)' \cdot u'$

$= \frac{1}{3} \cdot 5u^4 \cdot u'$

$= \frac{5}{3}(2x^5 + 6x^{-3})^4 \cdot (2 \cdot 5x^4 - 6 \cdot 3x^{-4})$

$= \frac{5}{3}(2x^5 + 6x^{-3})^4 \cdot (10x^4 - 18x^{-4})$

$= \frac{5}{3}(2x^5 + 6x^{-3})^4 \cdot 2 \cdot (5x^4 - 9x^{-4})$

$= \frac{10}{3}(2x^5 + 6x^{-3})^4 \cdot (5x^4 - 9x^{-4})$

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(14) $y = \ln(x^6 - 1) = \ln u$

$(\ln u)' = \frac{u'}{u} = \frac{(x^6 - 1)'}{x^6 - 1}$

$= \frac{6x^5}{x^6 - 1}$

(15) $y = \dfrac{1}{\sqrt[5]{x^3-1}} = \dfrac{1}{(x^3-1)^{1/5}} = (\underbrace{x^3-1}_{u})^{-1/5}$

$= f'(u) \cdot u'(x)$

$= (u^{-1/5})' \cdot u'(x)$

$= -\dfrac{1}{5} u^{-6/5} \cdot u'(x)$

$= -\dfrac{1}{5} \cdot (x^3-1)^{-6/5} \cdot (x^3-1)'$

$= -\dfrac{1}{5} (x^3-1)^{-6/5} \cdot (3x^2)$

$= -\dfrac{3x^2}{(x^3-1)^{6/5}} = -\dfrac{3x^2}{\sqrt[5]{(x^3-1)^6}}$

(16) $y = \cos(x^3-4) \Rightarrow \cos u$

$(\cos u)' = -u' \cdot \operatorname{sen} u$

$= -(x^3-4)' \cdot \operatorname{sen}(x^3-4)$

$= -(3x^2) \cdot \operatorname{sen}(x^3-4)$

$= -3x^2 \cdot \operatorname{sen}(x^3-4)$

(17) $y = (x^3 - 6)^5 \Rightarrow u^5$

$= f'(u) \cdot u'(x)$

$= (u^5)' \cdot u'(x)$

$= 5 \cdot u^4 \cdot u'(x)$

$= 5 \cdot (x^3 - 6)^4 \cdot (x^3 - 6)'$

$= 5 (x^3 - 6)^4 \cdot (3x^2)$

$= 15x^2 \cdot (x^3 - 6)^4$

(18) $y = 3x^2 + 5$

$y' = 3 \cdot 2x + 0$

$y' = 6x$

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