10. $\int_0^{\pi} (x + \operatorname{sen} x)\, dx$.

Pela propriedade 2, temos:

$$\int_0^{\pi} (x + \operatorname{sen} x)\, dx = \int_0^{\pi} x\, dx + \int_0^{\pi} \operatorname{sen} x\, dx$$

O TFC fornece, então:

$$\int_0^{\pi} (x + \operatorname{sen} x)\, dx = \frac{x^2}{2}\Big|_0^{\pi} + (-\cos x)\Big|_0^{\pi}$$

$$= \left(\frac{\pi^2}{2} - \frac{0^2}{2}\right) + \left(-\cos \pi - (-\cos 0)\right)$$

$$= \frac{\pi^2}{2} - \cos \pi + \cos 0$$

$$= \frac{\pi^2}{2} - (-1) + 1$$

$$= \frac{\pi^2}{2} + 1 + 1$$

$$= \frac{\pi^2}{2} + 2$$

6

(11) $\int_1^2 (e^x + 3)\,dx$

. Pelas propriedades das integrais, temos:

$$\int_1^2 (e^x+3)\,dx = \int_1^2 e^x\,dx + 3\int_1^2 dx$$

$$e^x\Big|_1^2 + 3x\Big|_1^2 = e^2 - e^1 + (3.2 - 3.1)$$

$$= e^2 - e + 3$$

(12) $\int_0^6 f(x)\,dx$, onde $f(x) = \begin{cases} x^2, & \text{se } 0 \leq x \leq 3 \\ 2x, & \text{se } 3 < x < 6 \\ 5, & \text{se } x = 6 \end{cases}$

Pela propriedade 3, temos:

$$\int_0^6 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx$$

$$= \int_0^3 x^2\,dx + \int_3^6 2x\,dx$$

$$= \frac{x^3}{3}\Big|_0^3 + \frac{\cancel{2}x^2}{\cancel{2}}\Big|_3^6 = \left(\frac{3^3}{3} - \frac{0^3}{3}\right) + (6^2 - 3^2)$$

$$= \frac{27}{3} - 0 + (36 - 9) = 9 + 27 = 36$$

(7)

(13) $\int (x^3 + \cos x)\, dx$

Pelas propriedades das integrais indefinidas e as fórmulas de integrais para as funções envolvidas, temos:

$$\int (x^3 + \cos x)\, dx = \int x^3 dx + \int \cos x \, dx$$

$$= \frac{x^4}{4} + C_1 + \operatorname{sen} x + C_2$$

Como C_1 e C_2 são constantes arbitrárias, podemos somá-las e obter uma nova constante: $C_1 + C_2 = C$.

Logo,

$$\int (x^3 + \cos x)\, dx = \frac{x^4}{4} + \operatorname{sen} x + C$$

(14) $\int (e^{2x} + x^2 + \operatorname{sen} x)\, dx$

$$= \int e^{2x} dx + \int x^2 dx + \int \operatorname{sen} x \, dx$$

$$= \frac{e^{2x}}{2} + \frac{x^3}{3} + (-\cos x) + C$$

$$= \frac{e^{2x}}{2} + \frac{x^3}{3} - \cos x + C$$

15 $\displaystyle\int \frac{dx}{x^2+1}$

Lembrando que

$$\frac{d}{dx}(\text{arc tg}\, x) = \frac{1}{1+x^2}, \text{ temos}$$

$$\int \frac{dx}{1+x^2} = \int \frac{1}{1+x^2}\, dx = \text{arc tg}\, x + C$$

16 $\displaystyle\int (7x^4 + \sec^2 x)\, dx$

Das propriedades das integrais indefinidas e da tabela de integrais imediatas, temos:

$$\int (7x^4 + \sec^2 x)\, dx = \int 7x^4 dx + \int \sec^2 x\, dx$$

$$= \frac{7x^5}{5} + C_1 + \text{tg}\, x + C_2 \quad , \text{ como } C_1 + C_2 = C, \text{ temos}$$

$$= \frac{7x^5}{5} + \text{tg}\, x + C$$

17) $\int \left(3e^x + \frac{1}{4x} - \operatorname{sen} x\right) dx$

Das propriedades da integral, temos:

$$= \int 3e^x\,dx + \int \frac{1}{4x}\,dx - \int \operatorname{sen} x\,dx$$

$$= 3\int e^x\,dx + \frac{1}{4}\int \frac{dx}{x} - \int \operatorname{sen} x\,dx$$

$$= 3 \cdot e^x + \frac{1}{4}\ln|x| - (-\cos x) + C$$

$$= 3e^x + \frac{1}{4}\ln|x| + \cos x + C$$

18) $\int_{\pi/4}^{\pi/2} \left(3e^x + \frac{1}{4x} - \operatorname{sen} x\right) dx$

Pelo exercício anterior e pelo TFC, temos:

$$3e^x + \frac{1}{4}\ln|x| + \cos x \Big|_{\pi/4}^{\pi/2} =$$

$$3\left(e^{\pi/2} - e^{\pi/4}\right) + \frac{1}{4}\left[\ln\left|\frac{\pi}{2}\right| - \ln\left|\frac{\pi}{4}\right|\right] + \left(\cancel{\cos\frac{\pi}{2}}^{\,0} - \cos\frac{\pi}{4}\right)$$

45°

$$3\left(e^{\pi/2} - e^{\pi/4}\right) + \frac{1}{4}\left(\ln\frac{\pi}{2} - \ln\frac{\pi}{4}\right) - \frac{\sqrt{2}}{2}$$

10

(19) $\int_1^4 (\sqrt{x} + x)\, dx$

$= \int_1^4 (x^{1/2} + x)\, dx$

Pelas propriedades da integral e pelo TFC temos

$= \int_1^4 x^{1/2} dx + \int_1^4 x\, dx$

$= \left.\frac{x^{3/2}}{3/2}\right|_1^4 + \left.\frac{x^2}{2}\right|_1^4 = \left.\frac{2}{3}\sqrt{x^3}\right|_1^4 + \left.\frac{x^2}{2}\right|_1^4$

$= \frac{2}{3}\left(\sqrt{4^3} - \sqrt{1^3}\right) + \left(\frac{4^2}{2} - \frac{1^2}{2}\right)$

$= \frac{2}{3}\left(\sqrt{64} - \sqrt{1}\right) + \left(\frac{16}{2} - \frac{1}{2}\right)$

$= \frac{2}{3}(8 - 1) + \frac{15}{2}$

$= \frac{2}{3} \cdot 7 + \frac{15}{2} = \frac{14}{3} + \frac{15}{2}$

$= \frac{28 + 45}{6} = \frac{73}{6}$