

Derivada da função inversa:

$$2) a) f(x) = x^3 - 8$$

$$y = x^3 - 8$$

$$y + 8 = x^3$$

$$x = \sqrt[3]{y+8} = (y+8)^{1/3}$$

$$f'(y) = \frac{1}{f'(x)} = \frac{1}{(x^3 - 8)'} = \frac{1}{3x^2}$$

Como $x = (y+8)^{1/3}$, temos:

$$f'(y) = \frac{1}{3((y+8)^{1/3})^2} =$$

$$= \frac{1}{3(y+8)^{2/3}}$$

$$= \frac{1}{3\sqrt[3]{(y+8)^2}}$$

$$b) f(x) = 3(x+2)^5 + 6 \Rightarrow f'(x) = 3 \cdot 5(x+2)^4$$

$$y = 3(x+2)^5 + 6$$

$$y - 6 = 3(x+2)^5$$

$$\frac{y-6}{3} = (x+2)^5$$

$$f'(x) = 15(x+2)^4$$

$$f'(y) = \frac{1}{f'(x)}$$

$$x+2 = \sqrt[5]{\frac{y-6}{3}}$$

$$f'(y) = \frac{1}{15(x+2)^4}$$

Sabemos que $x+2 = \left(\frac{y-6}{3}\right)^{1/5}$, temos:

$$f'(y) = \frac{1}{15 \cdot \left(\frac{y-6}{3}\right)^{4 \cdot 1/5}} = \frac{1}{15 \left(\frac{y-6}{3}\right)^{4/5}}$$

$$f'(y) = \frac{1}{15 \cdot \sqrt[5]{\left(\frac{y-6}{3}\right)^4}} = \frac{1}{15 \cdot \sqrt[5]{\left(\frac{3}{y-6}\right)^4}}$$

$$c) f(x) = 4 - 9x$$

$$f'(x) = -9$$

$$y = 4 - 9x$$

$$\sqrt{y+4} = 9x$$

$$x = \frac{y+4}{9}$$

$$f'(y) = \frac{1}{f'(x)} = -\frac{1}{9}$$

$$d) f(x) = \sqrt[3]{2x-1}$$

$$\Rightarrow f(x) = (2x-1)^{1/3}$$

$$f'(x) = \frac{1}{3}(2x-1)^{-2/3} \cdot 2$$

$$y = \sqrt[3]{2x-1}$$

$$\underline{y^3 = 2x-1}$$

$$f'(x) = \frac{2}{3(2x-1)^{2/3}}$$

$$f'(x) = \frac{2}{3\sqrt[3]{(2x-1)^2}}$$

$$f'(y) = \frac{1}{f'(x)}$$

$$f'(y) = \frac{1}{\frac{2}{3\sqrt[3]{(2x-1)^2}}}$$

mas $2x-1 = y^3$

$$f'(y) = 1 \cdot \frac{3\sqrt[3]{(2x-1)^2}}{2}$$

$$f'(y) = \frac{3\sqrt[3]{(y^3)^2}}{2} = \frac{3y^2}{2}$$

$$\textcircled{3} f(x) = x^5 + 4 \Rightarrow f'(x) = 5 \cdot x^4$$

$$f'(y) = \frac{1}{f'(x)}$$

$$\begin{aligned} y &= x^5 + 4 \\ x^5 &= y - 4 \\ x &= (y - 4)^{1/5} \end{aligned}$$

$$f'(y) = \frac{1}{5x^4}$$

$$f'(y) = \frac{1}{5((y-4)^{1/5})^4} = \frac{1}{5(y-4)^{4/5}}$$

$$f'(y) = \frac{1}{5(y-4)^{4/5}} = \frac{1}{5^5 \sqrt{(y-4)^4}}$$

$$a) (f^{-1})'(5) = \frac{1}{5^5 \sqrt{(5-4)^4}} = \frac{1}{5^5 \sqrt{1^4}} = \frac{1}{5}$$

$$b) (f^{-1})'(3) = \frac{1}{5^5 \sqrt{(3-4)^4}} = \frac{1}{5^5 \sqrt{(-1)^4}} = \frac{1}{5}$$

$$c) (f^{-1})'(4) = \frac{1}{5^5 \sqrt{(4-4)^4}} = \frac{1}{5 \cdot 0} = \frac{1}{0} = \text{undefined}$$

$$\begin{aligned} 1) (f^{-1})'(247) &= \frac{1}{5 \sqrt[5]{(247-4)^4}} = \frac{1}{5 \sqrt[5]{243^4}} \\ &= \frac{1}{5 \cdot 81} = \frac{1}{405} \end{aligned}$$