② c) $f(x) = 5x - 3 \quad , \quad x = -3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(-3+\Delta x) - f(-3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5(-3+\Delta x) - 3 - (5.(-3) - 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-15 + 5\Delta x - 3 - (-15 - 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-18 + 5\Delta x - (-18)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{-18} + 5\cancel{\Delta x} \cancel{+18}}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} 5 = 5$$

a) $f(x) = x^2 + x \quad , \quad x = 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(1+\Delta x)^2 + (1+\Delta x) - (1^2 + 1)^{=2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{1} + 2\Delta x + \Delta x^2 + \cancel{1} + \Delta x - \cancel{2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(3 + \Delta x)}{\cancel{\Delta x}} \Rightarrow \lim_{\Delta x \to 0} 3 + \Delta x = 3$$

b) $f(x) = \sqrt{x}$, $x = 4$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(4+\Delta x) - f(4)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{4+\Delta x} - \sqrt{4}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{4+\Delta x} - 2}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{4+\Delta x} - 2}{\Delta x} \cdot \frac{\sqrt{4+\Delta x} + 2}{(\sqrt{4+\Delta x} + 2)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(\cancel{\sqrt{4+\Delta x}})^{\cancel{2}} - 2^2}{\Delta x(\sqrt{4+\Delta x} + 2)}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{4} + \cancel{\Delta x} - \cancel{4}}{\cancel{\Delta x}(\sqrt{4+\Delta x} + 2)} =$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{1}{\sqrt{4+\Delta x} + 2} = \frac{1}{4}$$

c) $f(x) = \sqrt{x} \quad , \quad x = 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(3+\Delta x) - f(3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{3+\Delta x} - \sqrt{3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{3+\Delta x} - \sqrt{3}}{\Delta x} \cdot \frac{\sqrt{3+\Delta x} + \sqrt{3}}{\sqrt{3+\Delta x} + \sqrt{3}}$$

$$= \lim_{\Delta x \to 0} \frac{(\cancel{\sqrt{3+\Delta x}})^{\cancel{2}} - (\cancel{\sqrt{3}})^{\cancel{2}}}{\Delta x (\sqrt{3+\Delta x} + \sqrt{3})}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{3} + \cancel{\Delta x} - \cancel{3}}{\cancel{\Delta x} (\sqrt{3+\Delta x} + \sqrt{3})}$$

$$= \lim_{\Delta x \to} \frac{1}{\sqrt{3+\Delta x} + \sqrt{3}} = \frac{1}{\sqrt{3}+\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

d) $f(x) = \frac{1}{x} \quad , \quad x = 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(1) = \lim_{\Delta x \to 0} \frac{f(1+\Delta x) - f(1)}{\Delta x}$$

$$f'(1) = \lim_{\Delta x \to 0} \frac{\frac{1}{1+\Delta x} - \frac{1}{1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1 - (1+\Delta x)}{1+\Delta x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{\cancel{1} - \cancel{1} - \Delta x}{1+\Delta x}}{\Delta x}$$

$$\lim_{\Delta x \to 0} - \frac{\cancel{\Delta x}}{1+\Delta x} \cdot \frac{1}{\cancel{\Delta x}}$$

$$\lim_{\Delta x \to 0} - \frac{1}{1+\Delta x} = -1 \;//$$

② a) $f(x) = \frac{1}{x^2}$, $x = 2$

F

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(2) = \lim_{\Delta x \to 0} \frac{f(2+\Delta x) - f(2)}{\Delta x}$$

$$f'(2) = \lim_{\Delta x \to 0} \frac{\frac{1}{(2+\Delta x)^2} - \frac{1}{2^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{4+4\Delta x+\Delta x^2} - \frac{1}{4}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{4 - (4+4\Delta x+\Delta x^2)}{4(4+4\Delta x+\Delta x)^2}}{\frac{\Delta x}{1}}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{\cancel{4} - \cancel{4} - 4\Delta x + \Delta x^2}{4(4+4\Delta x+\Delta x^2)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-4\Delta x - \Delta x^2}{4(4+4\Delta x+\Delta x^2)} \cdot \frac{1}{\Delta x} = \frac{\cancel{\Delta x}(-4-\Delta x)}{4\cancel{\Delta x}(4+4\Delta x+\Delta x^2)}$$

$$= \lim_{\Delta x \to 0} \frac{-4-\Delta x^{0}}{16+16\Delta x+\Delta x^2} = -\frac{4}{16} = -\frac{1}{4}$$

g) $f(x) = 3x - 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{3(x+\Delta x) - 1 - (3x-1)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{3x} + 3\Delta x \cancel{- 1} \cancel{- 3x} \cancel{+ 1}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{3\cancel{\Delta x}}{\cancel{\Delta x}}$$

$$f'(x) = \lim_{\Delta x \to 0} 3 = 3 \,//$$

h) $f(x) = x^3$

$$\cancel{x^3} = (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$x^3 = (a+b)^3$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x}$$

$$= a^3 + 3a^2b + 3b^2a + b^3$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^3} + 3x^2 . \Delta x + 3 . x \Delta x^2 + \Delta x^3 - \cancel{x^3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\,(3x^2 + 3x\Delta x^{\,0} + \cancel{\Delta} x^{2\,0})}{\cancel{\Delta x}}$$

$$f'(x) = 3x^2 \;//$$

$$\sqrt{3x-4} \, . \, \sqrt{3x+4}$$

$$\sqrt{(3x)^2 - 4^2}$$

$$9x -$$

j) $f(x) = \sqrt{3x+4}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} = \frac{\sqrt{3.(x+\Delta x)+4} - \sqrt{3x+4}}{\Delta x}$$

$$= \frac{\sqrt{3x+3\Delta x+4} - \sqrt{3x+4}}{\Delta x} \cdot \frac{\sqrt{3x+3\Delta x+4} + \sqrt{3x+4}}{\sqrt{3x+3\Delta x+4} + \sqrt{3x+4}}$$

$$-(3x+4)$$

$$= \frac{\cancel{3x} + 3\cancel{\Delta x} + 4 - \cancel{3x} - 4}{\cancel{\Delta x}\left(\sqrt{3x+3\Delta x+4} + \sqrt{3x+4}\right)} = \frac{3}{\sqrt{3x+\cancel{\Delta x}+4} + \sqrt{3x+4}}$$

$$= \frac{3}{2\left(\sqrt{3x+4}\right)}$$

I

i) $f(x) = \dfrac{x}{x+1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\dfrac{x+\Delta x}{x+\Delta x+1} - \dfrac{x}{x+1}}{\Delta x}$$

$$= \frac{(x+1)(x+\Delta x) - x(x+\Delta x+1)}{(x+1)(x+\Delta x+1)} \cdot \frac{1}{\Delta x}$$

$$= \frac{\cancel{x^2} + \cancel{x\Delta x} + \cancel{x} + \Delta x - \cancel{x^2} - \cancel{x\Delta x} - \cancel{x}}{(x+1)(x+\Delta x+1)} \cdot \frac{1}{\Delta x}$$

$$= \frac{\Delta x}{x^2 + x\Delta x + x + x + \Delta x + 1}$$

$$= \frac{\cancel{\Delta x}}{\cancel{\Delta x}(x^2 + x\cancel{\Delta x} + 2x + \cancel{\Delta x} + 1)} = \frac{1}{x^2+2x+1}$$

$$= \frac{1}{(x+1)^2}$$

② k) $f(x) = \dfrac{x-3}{2x+4}$

K

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{(x+\Delta x)-3}{2(x+\Delta x)+4} - \dfrac{x-3}{2x+4}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{x+\Delta x-3}{2x+2\Delta x+4} - \dfrac{x-3}{2x+4}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{(2x+4)(x+\Delta x-3) - (x-3)(2x+2\Delta x+4)}{(2x+2\Delta x+4)(2x+4)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x^2 + 2x\Delta x - 6x + 4x + 4\Delta x - 12 - (2x^2 + 2x\Delta x + 4x - 6x - 6\Delta x - 12)}{(2x+2\Delta x+4)(2x+4)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x^2 + 2x\Delta x - 2x + 4\Delta x - 12 - (2x^2 + 2x\Delta x - 2x - 6\Delta x - 12)}{\Delta x(2x+2\Delta x+4)(2x+4)}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^2} + \cancel{2x\Delta x} - \cancel{2x} + 4\Delta x - \cancel{12} - \cancel{2x^2} - \cancel{2x\Delta x} + \cancel{2x} + 6\Delta x + \cancel{12}}{\Delta x(2x+2\Delta x+4)(2x+4)}$$

$$\lim_{\Delta x \to 0} \frac{10\Delta x}{\Delta x(2x+2\Delta x+4)(2x+4)} =$$

$$\lim_{\Delta x \to 0} \frac{10\cancel{\Delta x}}{\cancel{\Delta x}(2x+2\cancel{\Delta x}+4)(2x+4)} = \frac{10}{(2x+4)^2}$$

l) $f(x) = \sqrt{2x-5}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{2(x+\Delta x)-5} - \sqrt{2x-5}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{2x+2\Delta x-5} - \sqrt{2x-5}}{\Delta x} \cdot \frac{\sqrt{2x+2\Delta x-5} + \sqrt{2x-5}}{\sqrt{2x+2\Delta x-5} + \sqrt{2x-5}}$$

$$= \lim_{\Delta x \to 0} \frac{(\cancel{\sqrt{2x+2\Delta x-5}})^{\cancel{2}} - (\cancel{\sqrt{2x-5}})^{\cancel{2}}}{\Delta x\left(\sqrt{2x+2\Delta x-5} + \sqrt{2x-5}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{2x+2\Delta x-5-(2x-5)}{\Delta x\left(\sqrt{2x+2\Delta x-5} + \sqrt{2x-5}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x}+2\Delta x-\cancel{5}-\cancel{2x}+\cancel{5}}{\Delta x\left(\sqrt{2x+2\Delta x-5} + \sqrt{2x-5}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{2\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{2x+2\Delta x-5} + \sqrt{2x-5}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{2}{\sqrt{2x+2\underset{0}{\cancel{\Delta x}}-5} + \sqrt{2x-5}}$$

$$= \frac{2}{\sqrt{2x-5} + \sqrt{2x-5}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2x-5}}$$

$$f'(x) = \frac{1}{\sqrt{2x-5}}$$