Lista II

Derivadas Implícitas:

1) a) $x^3 + y^3 = 8$

$(x^3)' + (y^3)' = (8)'$

$3x^2 + 3y^2 \cdot y' = 0$

$3y^2 \cdot y' = -3x^2$

$y' = -\frac{3x^2}{3y^2}$

$y' = -\frac{x^2}{y^2}$

b) $4x^2 - 9y^2 = 17$

$(4x^2)' - (9y^2)' = (17)'$

$4 \cdot 2 \cdot x - 9 \cdot 2 y \cdot y' = 0$

$8x - 18y \cdot y' = 0$

$18y \cdot y' = 8x$

$y' = \frac{8x}{18y} \begin{matrix} :2 \\ :2 \end{matrix}$

$y' = \frac{4x}{9y}$

c) $\cos(x+y) + \operatorname{sen}(x+y) = \frac{1}{3}$

$$[\cos(x+y)]' + [\operatorname{sen}(x+y)]' = \left(\frac{1}{3}\right)'$$

$$(x+y)' \cdot (\cos'(x+y)) + (x+y)' \cdot (\operatorname{sen}'(x+y)) = 0$$

$$(1+y') \cdot (-\operatorname{sen}(x+y)) + (1+y') \cdot (\cos(x+y)) = 0$$

$$-\operatorname{sen}(x+y) - y' \cdot \operatorname{sen}(x+y) + \cos(x+y) + y'(\cos(x+y)) = 0$$

$$y' \cdot [-\operatorname{sen}(x+y) + \cos(x+y)] - \operatorname{sen}(x+y) + \cos(x+y) = 0$$

$$y' \cdot [-\operatorname{sen}(x+y) + \cos(x+y)] = \operatorname{sen}(x+y) - \cos(x+y)$$

$$y' = \frac{\operatorname{sen}(x+y) - \cos(x+y)}{-\operatorname{sen}(x+y) + \cos(x+y)}$$

$$y' = \frac{\cancel{\operatorname{sen}(x+y) - \cos(x+y)}}{-1\,[\cancel{\operatorname{sen}(x+y) - \cos(x+y)}]}$$

$$y' = \frac{1}{-1}$$

$$y' = -1 \;//$$

d) $tg(x+y) = 4$

$[tg(x+y)]' = (4)'$

$(x+y)' \cdot (tg'(x+y)) = 0$

$1 + y' \cdot [\sec^2(x+y)] = 0$

$\sec^2(x+y) + y'[\sec^2(x+y)] = 0$

$y'[\sec^2(x+y)] = -\sec^2(x+y)$

$y' = -\dfrac{\cancel{\sec^2(x+y)}}{\cancel{\sec^2(x+y)}}$

$y' = -1$

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e) $e^{\cos x} + e^{\operatorname{sen} y} = 1$

$(e^{\cos x})' + (e^{\operatorname{sen} y})' = (1)'$

$(\cos x)' \cdot e^{\cos x} + (\operatorname{sen} y)' \cdot e^{\operatorname{sen} y} \cdot y' = 0$

$-\operatorname{sen} x \cdot e^{\cos x} + y' \cdot \cos y \cdot e^{\operatorname{sen} y} = 0$

$y' \cdot \cos y \cdot e^{\operatorname{sen} y} = \operatorname{sen} x \cdot e^{\cos x}$

$$y' = \frac{\operatorname{sen} x \cdot e^{\cos x}}{\cos y \cdot e^{\operatorname{sen} y}}$$

f) $xy^2 + 2y^3 = x - 2y$

$(xy^2)' + (2y^3)' = x' - (2y)'$

$x' \cdot y^2 + (y^2)' \cdot x + 2 \cdot 3y^2 \cdot y' = 1 - 2y'$

$1 \cdot y^2 + 2xyy' + 6y^2 \cdot y' = 1 - 2y'$

$2xy \cdot y' + 6y^2 \cdot y' + 2y' = 1 - y^2$

$y'(2xy + 6y^2 + 2) = 1 - y^2$

$$y' = \frac{1 - y^2}{2xy + 6y^2 + 2}$$

g) $x^2y^2 + x\,\text{sen}\,y = 0$

$(x^2y^2)' + (x\,\text{sen}\,y)' = 0'$

$(x^2)'.y^2 + (y^2)'.x^2 + x'.\text{sen}\,y + x(\text{sen}\,y)'.y'$

$2x.y^2 + 2y.y'.x^2 + 1.\text{sen}\,y + x.(+\cos y).y'$

$2y.y'x^2 + x(\cos y).y' + 2xy^2 + \text{sen}\,y = 0$

$y'(2yx^2 + x\cos y) = -2xy^2 - \text{sen}\,y$

$$y' = \frac{-2xy^2 - \text{sen}\,y}{2x^2y + x\cos y}$$

h) $e^{x^2} + \ln y = 0$

$(e^{x^2})' + (\ln y)' = 0'$

$(x^2)'.e^{x^2} + \frac{1}{y}.y' = 0$

$2x.e^{x^2} + \frac{y'}{y} = 0$

$\frac{y'}{y} = -2.x\,e^{x^2}$

$y' = -2xye^{x^2}$

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j) $sen\left(\frac{x}{y}\right) = \frac{1}{2}$

$$\left[sen\left(\frac{x}{y}\right)\right]' = \left(\frac{1}{2}\right)'$$

$$\left(\frac{x}{y}\right)' \cdot \left[sen'\left(\frac{x}{y}\right)\right] = 0$$

$$\frac{1 \cdot y - y' \cdot x}{y^2} \cdot \left(\cos\left(\frac{x}{y}\right)\right) = 0$$

$$\frac{y \cdot \cos\left(\frac{x}{y}\right)}{y^2} - \frac{y' \cdot \cos\left(\frac{x}{y}\right) \cdot x}{y^2} = 0$$

$$\frac{y \cdot \cancel{\cos\left(\frac{x}{y}\right)}}{\cancel{y^2}} = \frac{y' \cancel{\cos\left(\frac{x}{y}\right)} \cdot x}{\cancel{y^2}}$$

$$y = y' \cdot x$$

$$y' = \frac{y}{x}$$

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i) $$\frac{2x+3y}{x^2+y^2} = 9$$

$$2x + 3y = 9(x^2 + y^2)$$

$$(2x)' + (3y)' = (9x^2)' + (9y^2)'$$

$$2 + 3y' = 9 \cdot 2x + 9 \cdot 2 \cdot y \cdot y'$$

$$2 + 3y' = 18x + 18y \cdot y'$$

$$2 - 18x = 18y \cdot y' - 3y'$$

$$2 - 18x = y'(18y - 3)$$

$$y' = \frac{2 - 18x}{18y - 3}$$